DIFFERENT ECCENTRIC DOMINATING SETS
WITH THEIR ENERGIES AND COUPLING DISTANCE
IN GRAPHS

State of the art of the research topic

The origin of graph theory can be traced back to August 26th, 1735 when Leonhard Euler (1707-1782) presented a paper 'Solutio problematis ad geometriam situs pertinentis’[10] in Petersburg Academy. This paper published in 1741 is considered to be the first work on graph theory. In 1878 for the first time, Silvester coined the term 'Graph' in his paper ”Chemistry and Algebra”[24] published in Nature journal.

Inspired by the queens problem of chess, in 1958 Claude Berge introduced 'coefficients of external stability' in his paper “Theory of graphs and its applications”[6], this concept later became famous as domination number. Oystein Ore coined the term 'dominating set' and 'domination number'[20]. “Optimal domination in graphs”[8] was the first paper published by Cockayne and Hedetniemi in 1977. $\gamma(G)$ was used to represent the domination number which eventually became the universally accepted notation[8]. An extensive survey on dominating sets can be seen in the two textbooks authored by Haynes, Hedetniemi and Slater[15, 14].

There are various distances like geodesic, detour[7], superior[18], monophonic[22], M-distance[26] and D-distance[4] introduced by different authors. There is a vast literature available on domination involving different distances[16, 23]. In this thesis, results related to domination based on distance related parameters are discussed.

In defence and other coveted services when there is indirect contact between two parties the mediators play a major role. Since too many carriers may compromise the secrecy of information keeping a common carrier is important. With this idea Anwar Alwardi, R. Rangarajan and Akram Alqesmah introduced injective domination of graphs[1] in the year 2018.

In daily life, we see mere contact does not lead to connection, relationship or friendship. Most of the time people align with others when they have equal status, equal wealth, equal qualifications or the same thoughts. Usually, we say that two integers are almost equal if $|a - b| < 1$. Using this concept equitable domination was introduced by Prof. E. Sampathkumar. If any two entities are connected to a common entity it may result in a
connection between them. For example, people often connect with their common friends. Inspired by this concept Anwar Alwardi, N.D Soner and Karam Ebadi introduced 'On the common neighbourhood domination number'\cite{2} in the year 2011. In network-based graphs the stability, integrity, connectivity and toughness of a vertex can be checked using a minimum dominating set which intersects all the other minimum dominating sets. It is easy to find the vulnerable vertices. Motivated by this theory 'Transversal domination in graphs'\cite{19} was introduced by Nayaka S.R, Anwar Alwardi and Puttaswamy in the year 2018. New ideas do not overshadow the old ones they rather generalize the range of existing phenomenon. Inspired by Janakiraman et al\cite{16} in this thesis an attempt has been made to define new variants of eccentric domination using injective\cite{1}, equitable\cite{23}, common neighbourhood\cite{3} and transversal\cite{19} dominations.

The field of Spectral graph theory is concerned with algebraic properties of graph related matrices. The study of matrices, eigenvalues and eigenvectors associated with graphs led to the evolution of spectral graph theory\cite{9}. The concept of energy arose in chemistry to approximate the total \( \pi \)-electron molecule of a conjugated hydrocarbon. Motivated by \( \pi \)-electron energy of molecular graphs Ivan Gutman\cite{11} introduced the concept of 'energy of a graph' in the year 1978. The energy of a graph \( G \) is the sum of absolute values of eigenvalues obtained by the adjacency matrix \( M(G) \).

Rajesh Kanna\cite{17} introduced the concept of minimum dominating energy of a graph. They also proved that minimum dominating energy depends on the dominating set. Tejaskumar R, A Mohamed Ismayil and Ivan Gutman\cite{25} introduced the concept of minimum eccentric dominating energy of graphs. Inspired by B. J. McClelland they gave bounds for crown, star, complete and cocktail party graphs.

**Definition of the problem**

The thesis entitled "Different eccentric dominating sets with their energies and coupling distance in graphs" is a sincere attempt to contribute to the wide area of graph theory, which mainly focuses on different eccentric dominations, various minimum eccentric dominating energies and coupling distance in graphs.

**Objectives and scope of the research work**

A distance based dominating set can be constructed by applying certain constraints
on a set of vertices involving certain distance parameters and establishing relation between
the characteristics of a vertex in a dominating set and another vertex in its complement
set.

1. To define various eccentricity based dominating sets like injective eccentric domi-
nating set (INED set), equitable eccentric dominating set (EQED set), common
neighbourhood eccentric dominating set (CNED set) and transversal eccentric domi-
inating set (TED set).

2. To obtain the bounds for injective eccentric domination number ($\gamma_{ined}$), equitable
eccentric domination number ($\gamma_{eqed}$), common neighbourhood eccentric domination
number ($\gamma_{cned}$) and transversal eccentric domination number ($\gamma_{ted}$) for different class
of graphs.

3. To define minimum INED energy ($E_{ined}$), minimum EQED energy ($E_{eqed}$), minimum
CNED energy ($E_{cned}$) and minimum TED energy ($E_{ted}$).

4. To analyze the properties of eigenvalues obtained from minimum INED matrix, minimum
EQED matrix, minimum CNED matrix and minimum TED matrix. In
addition to this find the corelation between these eigenvalues and their respective
energy related parameters.

5. To find the exact values of $E_{ined}$, $E_{eqed}$, $E_{cned}$ and $E_{ted}$ of standard graphs along
with their bounds.

6. To introduce the coupling distance in graphs, results related to the properties of
coupling distance parameters are analyzed.

Results and discussion

The results obtained in this thesis are with the reference to different eccentric domi-
nation, minimum eccentric dominating energies and coupling distance in graphs. The
organization of the thesis is as given below.

Chapter I: Introduction

This chapter is introductory in nature which covers the background and history of
the work presented, followed by some basic terminologies and definitions.

**Chapter II: Injective Eccentric Domination and its Energy of Graphs**

In this chapter, the concept of injective eccentric domination (INED) and its energy of graphs are introduced. Bounds on INED number $\gamma_{\text{ined}}(G)$ for some families of graphs are stated and proved. It is proved that, if a graph contains one or more pendant vertices then every minimum INED set contains at least one pendant vertex. The minimum INED energy $E_{\text{ined}}(G)$ is defined. The value of $E_{\text{ined}}(G)$ for the family of graphs like star, complete and cocktail party graphs are computed. Properties related to minimum INED eigenvalues are discussed. Lower and upper bounds for $E_{\text{ined}}(G)$ are established. The INED eigenvalues along with the minimum INED energy for standard graphs is tabulated. Minimum INED equienergetic graphs and minimum INED self equienergetic graphs are defined along with suitable examples.

**Chapter III: Equitable Eccentric Domination and its Energy of Graphs**

This chapter discusses about equitable eccentric domination (EQED) and its energy of graphs. The EQED number $\gamma_{\text{eqed}}(G)$ for different class of graphs are derived. Necessary and sufficient conditions for a minimal EQED set is proved and the minimum EQED energy $E_{\text{eqed}}(G)$ is discussed. The $E_{\text{eqed}}(G)$ for the complete, crown and cocktail party graphs are found. Properties related to minimum EQED eigenvalues are analyzed. Lower and upper bounds for $E_{\text{eqed}}(G)$ are discussed.

**Chapter IV: Common Neighbourhood Eccentric Domination and its Energy of Graphs**

Chapter IV deals with the common neighbourhood eccentric domination (CNED) and its energy of graphs. Theorems related to the exact values of CNED number $\gamma_{\text{cned}}(G)$ for some standard graphs are stated and proved along with some fundamental results. The minimum CNED energy $E_{\text{cned}}(G)$ for complete and cocktail party graphs are found and the properties of eigenvalues of $M_{\text{cned}}(G)$ is discussed. Bounds for minimum CNED energy of some standard graphs are obtained. The minimum CNED number, minimum CNED matrix, characteristic equation, eigenvalues and energy for some standard graphs
are tabulated.

Chapter V: Transversal Eccentric Domination and its Energy of Graphs

In this chapter, transversal eccentric domination (TED) and its energy of graphs are defined. Theorems related to finding the TED number $\gamma_{ted}(G)$ of different families of graphs are proved and some propositions on $\gamma_{ted}(G)$ are stated. The minimum TED energy of complete, crown and cocktail party graphs are found. The properties of minimum TED eigenvalues of $M_{ted}(G)$ are analyzed. Results related to the upper and lower bounds for minimum TED energy of standard graphs are presented.

Chapter VI: Coupling Distance in Graphs

In this chapter, the coupling distance of simple connected graphs is introduced. Different parameters of coupling distance like coupling eccentricity $Ce(v)$, coupling radius $Crad$, coupling diameter $Cdiam$, coupling center $CR$ and coupling periphery $CP$ are defined. Results related to coupling parameters of various class of graphs are obtained.

The content of this research work has been published in the following journals.

6. Riyaz Ur Rehman A, A Mohamed Ismayil and Ismail Naci Cangul, Common neig-
bourhood eccentric dominating and its energy of graphs, International Journal of

Communicated/accepted articles.
1. Riyaz Ur Rehman A and A Mohamed Ismayil, Injective eccentric domination in

Conclusion
This thesis mainly discusses about types of eccentric domination along with their re-
spective minimum eccentric dominating energies. In addition coupling distance in graphs
is introduced. Standard results related to different eccentric domination numbers along
with necessary and sufficient condition for various eccentric dominating sets are obtained.
Further more upper and lower bounds of different minimum eccentric dominating energies
are found.

References
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pages 547–556, 2011.
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(IJMSI), 2012.
vol. 2(9), Ashok Yakkaldevi, 2013.
1962.


Screenshot of the journal details.

**UGC-CARE List**

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Transversal eccentric domination in graphs

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Abstract
Eccentricity of a vertex vis a maximum among the shortest distances between the vertex v and all other vertices. A set D is called eccentric dominating if every vertex in its compliment has an eccentric vertex in the set D. A dominating set is transversal if the intersection of the set with all the minimum dominating sets is non-empty. Inspired by both the concepts we introduce transversal eccentric dominating (TED) set. An eccentric dominating set D is called a TED-set if it intersects with every minimum eccentric dominating set D'. We find the TED-number $\gamma_{ted}(G)$ of family of graphs, theorems related to their properties are stated and proved.

Keywords: Eccentricity, TED-set, TED-number.
2020 AMS subject classifications: 05C69. 1

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1Received on September 15, 2022. Accepted on December 15, 2022. Published on March 20, 2023. DOI: 10.23755/rm.v46i0.1043. ISSN: 1592-7415. eISSN: 2282-8214. ©Riyaz Ur Rehman et al. This paper is published under the CC-BY licence agreement.
1 Introduction

The classical queens problem in chess or the study of networks in electronics domination finds its application everywhere and plays a pivotal role in modern day science and technology. Domination is a vast arena in graph theory which is just not limited to adjacency between vertices belonging to the dominating set and its compliment. For a graph $G(V, E)$, a set $S \subseteq V$ is said to be a dominating set, if every vertex in $V-S$ is adjacent to some vertex in $S$. The domination number $\gamma_d(G)$ of a graph $G$ equals the minimum cardinality of an dominating set. There are many different invariants of domination. The concept of transversal domination in graphs was introduced by Nayaka S.R, Anwar Alwardi and Puttaswamy in 2018. A dominating set $D$ which intersects every minimum dominating set in $G$ is called a transversal dominating set. The minimum cardinality of a transversal dominating set is called the transversal domination number denoted by $\gamma_{td}(G)$. Geodesic being the shortest distance between any two vertices. The concept of shortest path has always intrigued the researchers in graph theory, operation research, computer science and other fields. There are many different types of distances in graphs, one such distance is eccentricity. The concept of eccentricity incorporated with a dominating set yields an eccentric dominating set. Eccentric domination was introduced by T. N. Janakiraman et al in 2010. The eccentricity $\epsilon(v)$ of $v$ is the distance to a vertex farthest from $v$. Thus, $\epsilon(v) = \max d(u,v): u \in V$. For a vertex $v$, each vertex at a distance $\epsilon(v)$ from $v$ is an eccentric vertex. Eccentric set of a vertex $v$ is defined as $E(v) = \{u \in G : d(u,v) = \epsilon(v)\}$. A set $D \subseteq V(G)$ is an eccentric dominating set if $D$ is a dominating set of $G$ and for every $v \in V-D$, there exists at least one eccentric vertex of $v$ in $D$. The eccentric domination number $\gamma_{ed}(G)$ of a graph $G$ equals the minimum cardinality of an eccentric dominating set. The main motive of this paper is to hybrid two different types of dominations and define a new domination variant. Inspired by this idea we combine transversal domination with eccentric domination. In this paper, we introduce transversal eccentric domination and calculate the TED-number of different graphs. Results related to TED-number of family of complete, star, path, cycle and wheel graphs are discussed. The upper TED-set, upper TED-number, lower TED-set and lower TED-number of different standard graphs are tabulated. For undefined terminologies refer the book graph theory by frank harary.

2 Transversal eccentric domination in graphs

Definition 2.1. An eccentric dominating set $S \subseteq V(G)$ is called a transversal eccentric dominating set (TED-set) if it intersects with every minimum ED-set $D'$ ie $S \cap D' \neq \emptyset$. 

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**Definition 2.2.** A TED-set $S$ is called a minimal TED-set if no proper subset of $S$ is TED-set.

**Definition 2.3.** The TED-number $\gamma_{ted}(G)$ of a graph $G$ is the minimum cardinality among the minimal TED-sets of $G$.

**Definition 2.4.** The upper TED-number $\Gamma_{ted}(G)$ of a graph $G$ is the maximum cardinality among the minimal TED-sets of $G$.

**Example 2.1.**

![Graph G](image)

Figure 2.1: Graph $G$

Consider the above example where the graph $G$ consists of 6 vertices and 9 edges.

(i) The dominating sets are $\{\wp_1, \wp_2\}$, $\{\wp_1, \wp_3\}$, $\{\wp_1, \wp_4\}$, $\{\wp_2, \wp_5\}$, $\{\wp_2, \wp_6\}$, $\{\wp_3, \wp_5\}$, $\{\wp_3, \wp_6\}$, $\{\wp_4, \wp_5\}$, $\{\wp_4, \wp_6\}$.

(ii) The minimum ED-sets are $\{\wp_1, \wp_2\}$, $\{\wp_3, \wp_5\}$, $\{\wp_4, \wp_6\}$.

(iii) The TED-sets are $\{\wp_1, \wp_5, \wp_6\}$, $\{\wp_2, \wp_3, \wp_4\}$.

**Observation 2.1.** For any graph $G$,

1. $\gamma(G) \leq \gamma_{ted}(G) \leq \gamma_{ted}(G) \leq \Gamma_{ted}(G)$.

2. $\gamma_{ted}(G) \leq n$ and $\Gamma_{ted}(G) \leq n$.

3. $V(G)$ is also a TED-set.

**Theorem 2.1.** For complete graph $K_n$, $\gamma_{ted}(K_n) = n$, $\forall n \geq 2$.

**Proof:** Let $V(K_n) = \{\wp_1, \wp_2, \ldots, \wp_n\}$. Since $\text{deg}(\wp_i) = n - 1 \ \forall \ \wp_i \in V(K_n)$ the eccentric vertex of $\wp_i$ is given by $E(\wp_i) = V - \{\wp_i\}$ and every single vertex dominates all other vertices. Since every vertex $\wp_i \in V$ forms an ED-set of the form $D_1 = \{\wp_1\}$, $D_2 = \{\wp_2\}$, $D_3 = \{\wp_3\}$, $\ldots$, $D_n = \{\wp_n\}$. The vertex set $V$ is the only set which forms a TED-set, since $V(K_n) \cap D_i \neq \emptyset$ where $i = 1, 2, 3, \ldots n$ and $D_i$ is any ED-set.
Theorem 2.2. For star graph $S_n$, $\gamma_{ted}(S_n) = 2 \forall n \geq 3$.

Proof: Let $V(S_n) = \{\varphi_1, \ldots, \varphi_i, \ldots, \varphi_n\}$ where $deg(\varphi_i) = n - 1$ where $\varphi_i$ is the central vertex and $deg(\varphi_j) = 1$ where $\varphi_j$ is a pendant vertex of star graph $S_n$. $E(\varphi_i) = V - \{\varphi_i\}$ and $E(\varphi_j) = V - \{\varphi_i, \varphi_j\}$. The central vertex $\varphi_i$ forms a dominating set $\{\varphi_i\}$ but it is not an ED-set for any $\varphi_j \in V - D$, $E(\varphi_j) \notin D$. But $D = \{\varphi_i, \varphi_j\}$ forms an ED-set, then for $S_3$ we have 3 ED-sets which forms the minimum ED-sets and for any star graph $S_n$, $\forall n \geq 4$, we have $(n - 1)$ ED-sets which forms the minimum ED-sets $D_1 = \{\varphi_i, \varphi_1\}, D_2 = \{\varphi_i, \varphi_2\}, D_3 = \{\varphi_i, \varphi_3\}, \ldots D_n = \{\varphi_i, \varphi_n\}$. Any minimum ED-set $D = \{\varphi_i, \varphi_j\}$ also forms a TED-set, since $D \cap \{\varphi_i, \varphi_j\} = \{\varphi_i\} \neq \emptyset$. Therefore $\gamma_{ted}(S_n) = 2 \forall n \geq 3$.

Theorem 2.3. For path graph $P_n$, $\gamma_{ted}(P_n) = \lceil \frac{n+1}{3} \rceil + 1, \forall n \geq 2$.

Proof: Let the vertices of $P_n$ be given by $V(P_n) = \{\varphi_1, \varphi_2, \ldots, \varphi_n\}$. Every path $P_n$ contains two pendant vertices $\{\varphi_1, \varphi_n\}$. For any vertex $\varphi_k \in V(P_n)$ the eccentric vertex of $\varphi_k$ is $E(\varphi_k) = \{\varphi_1\}$ or $\{\varphi_n\}$ where $n$ is even. If $n$ is odd then $E(\varphi_k) = \{\varphi_1\}$ or $\{\varphi_n\}$ but if $\varphi_k$ is a vertex equidistant from both the pendant vertices then $\varphi_k = \frac{n+1}{2}, E(\varphi_k) = \{\varphi_1, \varphi_n\}$. For any path $P_n$, $[\frac{n}{3}]$ set of vertices can dominate all the vertices of $P_n$. Similarly a set $D$ whose cardinality is $[\frac{n+1}{3}] + 1$ will eccentric dominate all the vertices of $P_n$. By the definition of TED-set, a set $D$ should intersect all the minimum ED-set. An ED-set $D$ will intersect all the minimum ED-sets. Therefore every minimum ED-set is a TED-set. Therefore $\gamma_{ted}(P_n) = \gamma_{ted} = [\frac{n+1}{3}] + 1$.

Theorem 2.4. For cycle graph $C_n$ where $n \geq 3$

$$\gamma_{ted}(C_n) = \begin{cases} 5, & \text{for } n = 8 \\ [\frac{n+1}{3}] + 1, & \text{otherwise} \end{cases}$$

Proof: Case(i): For $C_8$, the set $D = \{\varphi_i, \varphi_j, \varphi_k, \varphi_l\}$ whose cardinality is $[\frac{n+1}{3}] + 1 = 4$ does not form a TED-set which is an exception from case(i). Adding a vertex to $D$ is of the form $\{\varphi_i, \varphi_j, \varphi_k, \varphi_l, \varphi_m\}$ whose cardinality is five will increasing the cardinality of $D$. Here every vertex in $V(C_8) - D$ has an eccentric vertex in $D$ and $D$ is also dominating set which intersects all the minimum dominating sets of $C_8$. Therefore $\gamma_{ted}(C_8) = 5$.

Case(ii): For a cycle graph $C_n$, if $n$ is even and $n \neq 8$ then every vertex $\varphi_i \in V(C_n)$ has a unique eccentric vertex $\varphi_i$, $E(\varphi_i) = \{\varphi_j | \varphi_j \in V(C_n)\}$. $E(\varphi_i)$ is at a distance of $\frac{n}{2}$ edges from $\varphi_i$ for an even cycle. If $n$ is odd then every vertex $\varphi_i$ has two eccentric vertices. $E(\varphi_i) = \{\varphi_j, \varphi_k | \varphi_j, \varphi_k \in V(C_n)\}$. $E(\varphi_i)$ is at a distance of $\frac{n}{2}$ edges from $\varphi_i$ for odd cycle. Every single vertex $\varphi_i$ can dominate itself and two vertices adjacent to it. Therefore for any cycle $C_n$, $[\frac{n}{3}]$ set of vertices forms the dominating set. Here we see that any set $D = \{\varphi_1, \varphi_2, \ldots, \varphi_i\}$ which has the cardinality of the form $[\frac{n+1}{3}] + 1$ forms a dominating set as well.
as an ED-set. Since $D$ whose cardinality is $\lceil \frac{n+1}{3} \rceil + 1$ intersects every minimum ED-set of cardinality $\gamma_{ed}(C_n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ \lceil \frac{n}{3} \rceil \text{ or } \lceil \frac{n}{3} \rceil + 1, & \text{if } n \text{ is odd} \end{cases}$. $D$ forms a TED-set. Hence $\gamma_{ted}(C_n) = \lceil \frac{n+1}{3} \rceil + 1$.

**Theorem 2.5.** For wheel graph $W_n$ where $n \geq 4$, $a \geq 1$

$\gamma_{ted}(W_n) = \begin{cases} 3, & \text{for } n = (6a - 1), (6a) \text{ or } (6a + 1) \\ 4, & \text{for } n = (6a - 2), (6a + 2) \text{ or } (6a + 3) \end{cases}$

**Proof:** Case(i): If $n = 6a - 1$, $6a$ and $6a + 1$, the wheel graphs are of the form $W_5, W_6, W_7, W_{11}, W_{12}, W_{13}, W_{17}, W_{18}, W_{19}, \ldots W_{6a-1}, W_{6a}, W_{6a+1}$. Let $\varphi_c$ be the central vertex of wheel graph, $\deg(\varphi_c) = n - 1$. Therefore $\varphi_c$ has $n - 1$ eccentric vertices, $|E(\varphi_c)| = n - 1$. Let $\varphi_i$ be the non-central vertex, $\deg(\varphi_i) = 3$. Then closed neighbourhood of $\varphi_i$, $N[\varphi_i] = 4$. Therefore $\varphi_i$ has $n - 4$ eccentric vertices, $|E(\varphi_i)| = n - 4$. Let $G = \{\varphi_c\}$ forms the only dominating set of cardinality one, but not an ED-set. Other than $W_5$ and $W_7$ every other wheel graph has an ED-set $D = \{\varphi_c, \varphi_x, \varphi_y\}$ where $\varphi_c, \varphi_x, \varphi_y \in V(W_n)$ forms an ED-set and for every $v \in V(W_n) - D$ there exists a vertex $\varphi_c, \varphi_x$ or $\varphi_y$ in $D$ such that $E(v) = \varphi_c$ or $\varphi_x$ or $\varphi_y$ and $D = \{\varphi_c, \varphi_x, \varphi_y\}$ forms a TED-set, since $D$ intersects every minimum ED-set. Therefore $|D| = 3$, $\gamma_{ted}(W_n) = 3$ for $n = 6a - 1, 6a, 6a + 1$.

Case(ii): If $(6a - 2), (6a + 2)$ and $(6a + 3)$, then the wheel graphs are of the form $W_4, W_8, W_9, W_{10}, W_{14}, W_{15}, W_{16}, \ldots W_{6a-2}, W_{6a+2}, W_{6a+3}$. For $W_4$, $\gamma_{ted}(W_4) = 4$. Since $W_4$ is $K_4$ which is complete graph (by theorem 2.1). Similar to case(i), $\varphi_c$ is the central vertex of wheel graph and $\varphi_j$ is the non-central vertex, $|E(\varphi_c)| = n - 1$ and $|E(\varphi_j)| = n - 4$. Similar to case(i) the only unique dominating set $D = \{\varphi_c\}$ whose cardinality is one does not form an ED-set. But a set $D = \{\varphi_c, \varphi_x, \varphi_y\}$ containing three vertices forms an ED-set, since every vertex $\varphi_i \in V(W_n) - D$ has an eccentric vertex in $D$ i.e., $E(\varphi_i) = \varphi_c, \varphi_x$ or $\varphi_y$. But $D = \{\varphi_c, \varphi_x, \varphi_y\}$ whose cardinality is three does not form a TED-set since it does not intersect every minimum ED-set. But an addition of vertex $\varphi_z$ to the same set gives us a set $D = \{\varphi_c, \varphi_x, \varphi_y, \varphi_z\}$ whose cardinality is four forms an ED-set and it intersects every minimum ED-set of cardinality three, thus becoming TED-set. Therefore $\gamma_{ted}(W_n) = 4$ for $n = (6a - 2), (6a + 2)$ and $(6a + 3)$.

**Proposition 2.1.** For any graph $G$,

1. $\gamma_{ted}(G) \geq \lceil \frac{2n-q}{4} \rceil$.
2. $\gamma_{ted}(G) \geq \frac{\text{diam}(G)+1}{4}$.
3. $\gamma_{ted}(G) \leq \lceil \frac{p \Delta(G)}{\delta} \rceil$. 

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4. $\gamma_{ted}(G) \geq \left\lceil \frac{p}{1+\Delta(G)} \right\rceil$.

5. $\gamma_{ted}(G) \leq \lceil n + \Delta(G) - \sqrt{2q} \rceil$.

The transversal eccentric dominating set, $\gamma_{ted}(G)$, upper transversal eccentric dominating set and $\Gamma_{ted}(G)$ of standard graphs are tabulated.

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<td>3</td>
<td>${\varphi_1, \varphi_2, \varphi_3, \varphi_4}$</td>
<td>4</td>
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</table>
Transversal eccentric domination in graphs

<table>
<thead>
<tr>
<th>Graph</th>
<th>Figure</th>
<th>D - Minimum TED set.</th>
<th>S - Upper TED set.</th>
<th>$\gamma_{ted}(G)$</th>
<th>$\Gamma_{ted}(G)$</th>
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<tr>
<td>Paw graph</td>
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<td>${\varphi_1, \varphi_3}$, ${\varphi_2, \varphi_3}$</td>
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<td>Bull graph</td>
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<td>Butterfly graph</td>
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<td>${\varphi_1, \varphi_2, \varphi_3}$, ${\varphi_1, \varphi_2, \varphi_4}$, ${\varphi_1, \varphi_3, \varphi_4}$, ${\varphi_2, \varphi_3, \varphi_4}$</td>
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<td>Banner graph</td>
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<td>${\varphi_2, \varphi_5}$</td>
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<td>Fork graph</td>
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<td>${\varphi_1, \varphi_2, \varphi_4}$, ${\varphi_1, \varphi_3, \varphi_4}$</td>
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<td>(3,2)-Tadpole graph</td>
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<td>(4,1)-Lollipop graph</td>
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<tr>
<td>Graph</td>
<td>Figure</td>
<td>D - Minimum TED set. $</td>
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<td>= \gamma_{ad}(G)$</td>
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<tr>
<td>House graph</td>
<td><img src="image" alt="House Graph" /></td>
<td>${v_1, v_3, v_5},$ ${v_1, v_2, v_5},$ ${v_1, v_2, v_3},$ ${v_2, v_3, v_4},$ ${v_2, v_3, v_5},$ ${v_2, v_4, v_5},$ ${v_2, v_4, v_5},$ ${v_1, v_4, v_5}. $</td>
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<tr>
<td>House X graph</td>
<td><img src="image" alt="House X Graph" /></td>
<td>${v_1, v_2},$ ${v_1, v_3},$ ${v_1, v_4},$ ${v_1, v_5}. $</td>
<td>2</td>
<td>${v_2, v_3, v_4, v_5}. $</td>
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<tr>
<td>Gem graph</td>
<td><img src="image" alt="Gem Graph" /></td>
<td>${v_1, v_2, v_3},$ ${v_1, v_2, v_4},$ ${v_1, v_2, v_5},$ ${v_1, v_3, v_4},$ ${v_1, v_3, v_5},$ ${v_2, v_3, v_4},$ ${v_2, v_3, v_5},$ ${v_2, v_4, v_5}. $</td>
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<tr>
<td>Dart graph</td>
<td><img src="image" alt="Dart Graph" /></td>
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<td>Cricket graph</td>
<td><img src="image" alt="Cricket Graph" /></td>
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<td>Pentatope graph</td>
<td><img src="image" alt="Pentatope Graph" /></td>
<td>${v_1, v_2, v_3, v_4, v_5}. $</td>
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<tr>
<td>Johnson solid skeleton 12 graph</td>
<td><img src="image" alt="Johnson Solid Skeleton 12 Graph" /></td>
<td>${v_1, v_2, v_3}. $</td>
<td>2</td>
<td>${v_1, v_2, v_3, v_4, v_5},$ ${v_2, v_3, v_4, v_5},$ ${v_2, v_3, v_4, v_5}. $</td>
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<tr>
<td>Cross graph</td>
<td><img src="image" alt="Cross Graph" /></td>
<td>${v_1, v_3, v_5},$ ${v_2, v_3, v_5},$ ${v_2, v_3, v_6},$ ${v_3, v_4, v_5},$ ${v_3, v_5, v_6}. $</td>
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<td>5</td>
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</tbody>
</table>
Transversal eccentric domination in graphs

| Graph         | Figure | D - Minimum TED set. \(|D| = \gamma_{\text{ted}}(G)\) | \(\gamma_{\text{ted}}(G)\) | S - Upper TED set. \(|S| = \Gamma_{\text{ted}}(G)\) | \(\Gamma_{\text{ted}}(G)\) |
|---------------|--------|---------------------------------|-----------------|---------------------------------|-----------------|
| Net graph     | ![Net Graph](image) | \(\{\varphi_1, \varphi_2, \varphi_6\}\), \(\{\varphi_1, \varphi_2, \varphi_5\}\), \(\{\varphi_2, \varphi_3, \varphi_5\}\). | 3               | \(\{\varphi_1, \varphi_2, \varphi_4, \varphi_6\}\), \(\{\varphi_1, \varphi_3, \varphi_5, \varphi_6\}\), \(\{\varphi_2, \varphi_5, \varphi_6\}\). | 4               |
| Fish graph    | ![Fish Graph](image) | \(\{\varphi_2, \varphi_3\}\), \(\{\varphi_2, \varphi_5\}\). | 2               | \(\{\varphi_1, \varphi_2, \varphi_4, \varphi_5, \varphi_6\}\). | 5               |
| A graph       | ![A Graph](image) | \(\{\varphi_1, \varphi_3, \varphi_6\}\), \(\{\varphi_2, \varphi_3, \varphi_5\}\). | 3               | \(\{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_6\}\), \(\{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\}\). | 5               |
| R graph       | ![R Graph](image) | \(\{\varphi_1, \varphi_2, \varphi_3\}\), \(\{\varphi_2, \varphi_3, \varphi_4\}\), \(\{\varphi_2, \varphi_3, \varphi_5\}\). | 3               | \(\{\varphi_1, \varphi_3, \varphi_4, \varphi_5, \varphi_6\}\). | 5               |
| 4-polynomial graph | ![4-polynomial Graph](image) | \(\{\varphi_2, \varphi_4\}\). | 2               | \(\{\varphi_1, \varphi_2, \varphi_3, \varphi_5, \varphi_6\}\), \(\{\varphi_1, \varphi_2, \varphi_4, \varphi_5, \varphi_6\}\). | 5               |
| Octahedral graph | ![Octahedral Graph](image) | \(\{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}\), \(\{\varphi_1, \varphi_2, \varphi_3, \varphi_5\}\), \(\{\varphi_1, \varphi_2, \varphi_3, \varphi_6\}\), \(\{\varphi_1, \varphi_2, \varphi_4, \varphi_5\}\), \(\{\varphi_1, \varphi_2, \varphi_4, \varphi_6\}\), \(\{\varphi_1, \varphi_2, \varphi_5, \varphi_6\}\), \(\{\varphi_1, \varphi_3, \varphi_4, \varphi_5\}\), \(\{\varphi_1, \varphi_3, \varphi_4, \varphi_6\}\), \(\{\varphi_1, \varphi_3, \varphi_5, \varphi_6\}\), \(\{\varphi_2, \varphi_3, \varphi_4, \varphi_5\}\), \(\{\varphi_2, \varphi_3, \varphi_4, \varphi_6\}\), \(\{\varphi_2, \varphi_3, \varphi_5, \varphi_6\}\). | 4               | \(\{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}\), \(\{\varphi_1, \varphi_2, \varphi_3, \varphi_5\}\), \(\{\varphi_1, \varphi_2, \varphi_3, \varphi_6\}\), \(\{\varphi_1, \varphi_2, \varphi_4, \varphi_5\}\), \(\{\varphi_1, \varphi_2, \varphi_4, \varphi_6\}\), \(\{\varphi_1, \varphi_2, \varphi_5, \varphi_6\}\), \(\{\varphi_1, \varphi_3, \varphi_4, \varphi_5\}\), \(\{\varphi_1, \varphi_3, \varphi_4, \varphi_6\}\), \(\{\varphi_1, \varphi_3, \varphi_5, \varphi_6\}\), \(\{\varphi_2, \varphi_3, \varphi_4, \varphi_5\}\), \(\{\varphi_2, \varphi_3, \varphi_4, \varphi_6\}\), \(\{\varphi_2, \varphi_3, \varphi_5, \varphi_6\}\). | 4               |

### 3 Conclusions

In this paper TED-set of a graph is defined. Theorems related to find the TED-number of different family of graphs are stated and proved. The upper and lower
| Graph          | Figure | D - Minimum TED set. | $|D| = \gamma_{ted}(G)$ | S - Upper TED set. | $|S| = \Gamma_{ted}(G)$ | $\Gamma_{ted}(G)$ |
|---------------|--------|----------------------|-------------------------|-------------------|-------------------------|------------------|
| 3-prism graph | ![Triangle](image) | $\{p_1, p_5, p_6\}, \{p_2, p_3, p_4\}$ | 3                      | $\{p_1, p_2, p_4, p_5\}, \{p_1, p_3, p_4, p_5\}, \{p_1, p_3, p_5, p_6\}, \{p_2, p_3, p_5, p_6\}, \{p_2, p_4, p_5, p_6\}$ | 4               |

TED-number along with their respective sets of different standard graphs are tabulated. In future the comparative study of TED-set with eccentric dominating set will be done. The properties of a TED-set related to graph operations such as union, intersection, join and product of graphs will be explored.

**References**


Equitable eccentric domination in graphs

Riyaz Ur Rehman A*
A Mohamed Ismayil†

Abstract

In this paper, we define equitable eccentric domination in graphs. An eccentric dominating set \( S \subseteq V(G) \) of a graph \( G(V, E) \) is called an equitable eccentric dominating set if for every \( v \in V - S \) there exist at least one vertex \( u \in V \) such that \( |d(v) - d(u)| \leq 1 \) where \( vu \in E(G) \). We find equitable eccentric domination number \( \gamma_{eqed}(G) \) for most popular known graphs. Theorems related to \( \gamma_{eqed}(G) \) have been stated and proved.

Keywords: eccentricity, equitable domination number, equitable eccentric domination number.

2020 AMS subject classifications: 05C69. 1

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1Received on September 15, 2022. Accepted on December 15, 2022. Published online on January 10, 2023. DOI: 10.23755/rm.v41i0.802. ISSN: 1592-7415, eISSN: 2282-8214. ©The Authors. This paper is published under the CC-BY licence agreement.
1 Introduction

A graph is a representation of a pair of sets $(V, E)$, where $V$ is the set of vertices and $E$ is the set of edges which are connecting the pair of vertices. Graph theory has its application in many fields such as computation, social and natural science etc. Any problems of mathematics, science and engineering can be represented in the form of a graph. The concept of graph theory was first introduced by Leonard Euler in the year 1736. He created the first graph as a solution to solve the problem of seven bridges of Konigsberge built across the pregel river of prussia. Graph theory has experienced tremendous growth, the main reason for this phenomena is applicability of graph theory in different disciplines. Graph theory becomes interesting because graphs can be used to model situations that occur in real world problems. These problems can be studied with the aid of graphs.


The concept of eccentricity by T.N. Janakiraman et al has inspired researchers which has led to many invariants of eccentric dominations in graphs. Some of the extended eccentric dominations are accurate eccentric domination[7] and equal eccentric domination[8]. The concept of geodesic distance is very important. The existing eccentric domination only highlighted the idea based on an eccentric vertex and its domination. The proposed equitable eccentric domination was mainly necessary because it highlights the properties of a vertex in a graph, it considers the connectivity between the vertices where the difference between their vertex degrees is less than or equal to one. Equitable domination when incorporated with eccentric domination yields equitable eccentric domination which concentrates on the vertex degree, geodesic distance, eccentricity, eccentric vertex and domination. In this paper, we introduce equitable eccentric domination in graphs. We
find equitable eccentric dominating set, equitable eccentric domination number $\gamma_{eqed}(G)$, upper equitable eccentric dominating set and upper equitable eccentric domination number $\Gamma_{eqed}(G)$ of different standard graphs. For undefined graph terminologies refer the book 'Graph theory' by Frank Harary[5].

2 Preliminaries

Definition 2.1 ([11]). Let $G$ be a graph with the vertex set $V$. A subset $D$ of $V$ is a dominating set for $G$ when every vertex not in $D$ is the endpoint of some edge from a vertex in $D$.

Definition 2.2 ([10]). Let $\gamma(G)$ (called the domination number) and $\Gamma(G)$ (called the upper domination number) be the minimum cardinality and the maximum cardinality of a minimal dominating set of $G$, respectively.

Definition 2.3 ([6]). The degree $\deg(v)$ of $v$ is the number of edges incident with $v$.

Definition 2.4 ([9]). The eccentricity $e(v)$ of $v$ is the distance to a vertex farthest from $v$. Thus, $e(v) = \max\{d(u, v) : u \in V\}$. For a vertex $v$, each vertex at a distance $e(v)$ from $v$ is an eccentric vertex. Eccentric set of a vertex $v$ is defined as $E(v) = \{u \in V(G) / d(u, v) = e(v)\}$.

Definition 2.5 ([9]). The radius $r(G)$ is the minimum eccentricity of the vertices, whereas the diameter $\text{diam}(G)$ is the maximum eccentricity.

Definition 2.6 ([9]). $v$ is a central vertex if $e(v) = r(G)$. The center $C(G)$ is the set of all central vertices. $v$ is a peripheral vertex if $e(v) = \text{diam}(G)$. The periphery $P(G)$ is the set of all peripheral vertices.

Definition 2.7 ([9]). A set $D \subseteq V(G)$ is an eccentric dominating set if $D$ is a dominating set of $G$ and for every $v \in V - D$, there exists at least one eccentric point of $v$ in $D$. If $D$ is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set. An eccentric dominating set $D$ is a minimal eccentric dominating set if no proper subset $D'' \subseteq D$ is an eccentric dominating set.

Definition 2.8 ([9]). The eccentric domination number $\gamma_{ed}(G)$ of a graph $G$ equals the minimum cardinality of an eccentric dominating set. That is, $\gamma_{ed}(G) = \min|D|$, where the minimum is taken over $D$ in $G$, where $D$ is the set of all minimal eccentric dominating sets of $G$.

Definition 2.9 ([4]). A subset $D$ of $V$ is called an equitable dominating set if for every $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality of such a dominating set is denoted by $\gamma^e$ and is called the equitable domination number of $G$. 
3 Equitable eccentric domination in graphs

In this section we introduce equitable eccentric domination, theorems related to equitable eccentric domination number of family of graphs are stated and proved.

**Definition 3.1.** An eccentric dominating set \( S \subseteq V(G) \) is called an equitable eccentric dominating set (EQED-set) if for every \( v \in V - S \) there exist at least one vertex \( u \in S \) such that \( vu \in E(G) \) and \( |d(v) - d(u)| \leq 1 \).

**Definition 3.2.** An equitable eccentric dominating set \( S \) is called a minimal equitable eccentric dominating set if no proper subset of \( S \) is equitable eccentric dominating set.

**Definition 3.3.** The equitable eccentric domination number \( \gamma_{eqed}(G) \) of a graph \( G \) is the minimum cardinality among the minimal equitable eccentric dominating sets of \( G \).

**Definition 3.4.** The upper equitable eccentric domination number \( \Gamma_{eqed}(G) \) of a graph \( G \) is the maximum cardinality among the minimal equitable eccentric dominating sets of \( G \).

**Example 3.1.**

Consider the graph \( G \) consists of 6 vertices given in figure 2.1. Here the dominating set is \( S = \{v_1, v_4\} \) but not eccentric dominating set since \( E(v_3) = \{v_2, v_6\} \) not in \( S \). The eccentric dominating set is \( S = \{v_1, v_6\} \) but not equitable eccentric dominating set since \( |d(v_4) - d(v_6)| = 2 \). The equitable eccentric dominating set is \( S = \{v_1, v_2, v_6\} \).

**Remark 3.1.** For any path \( P_n \) where \( n \geq 3 \),
1. Every minimum EQED-set contains the pendant vertices.
2. If \( D_1, D_2, D_3 \) are minimum EQED-sets of paths \( P_{n-1}, P_n, P_{n+1} \) consecutively where \( n = 3k \) and \( k > 1 \). Then \( |D_1| = |D_2| = |D_3| \). Therefore for \( k = 2 \),
   \( \gamma_{eqed}(P_5) = \gamma_{eqed}(P_6) = \gamma_{eqed}(P_7) = 3 \).

**Theorem 3.1.** For complete graph \( K_n \), \( \gamma_{eqed}(K_n) = 1, \forall n \geq 2 \).
Equitable eccentric domination

**Proof.** In a complete graph $K_n$ all the vertices are eccentric vertices to each other. If $v \in V(K_n)$ then the eccentric vertex $E(v) = V(K_n) - \{v\}$ and every singleton set forms a dominating set. For every vertex $v \in D \ni u \in V(K_n) - D \ni |\text{deg}(u) - \text{deg}(v)| \leq 1$ where $uv \in E(K_n)$. Therefore every single vertex of $K_n$ is an EQED-set. Hence $\gamma_{eqed}(K_n) = 1$.

**Theorem 3.2.** For path graph $P_n$ where $n > 1$,

$$\gamma_{eqed}(P_n) = \begin{cases} 1, & \text{for } n = 2 \\ \left\lceil \frac{n+1}{3} \right\rceil + 1, & \forall \ n \geq 3 \end{cases}$$

**Proof.** Case(i): For a path $P_2$, $V(P_2) = \{v_1, v_2\}$. Both the vertices are eccentric vertices to each other. Therefore $D = \{v_1\}$ or $\{v_2\}$ and $|\text{deg}(v_2) - \text{deg}(v_1)| = 0$, where $v_1v_2 \in E(P_2)$. Hence $\gamma_{eqed}(P_2) = 1$.

Case(ii): For a path $P_n$ where $n \geq 3$. The pendant or end vertices of the path form the eccentric vertices ie, if $V(P_n) = \{v_1, v_2, \ldots, v_n\}$, $E(v_1) = \{v_n\}$ and $E(v_n) = \{v_1\}$. $E(v_i) = \{v_1\}$ or $\{v_n\}$ for any $v_i \in V(P_n)$ where $n$ is even. If $n$ is odd then $E(v_i) = \{v_1\}$ or $\{v_n\}$. For $P_n$ where $n'$ is odd, the central vertex $v_i$ has two eccentric vertices ie, $E(v_i) = \{v_1, v_n\}$. Degree of end vertices is 1 and degree of all the intermediate vertices is 2. The EQED-set contains both the pendant vertices. Both $v_1$ and $v_n$ being pendant vertices dominate the vertices adjacent to them and the minimum dominating set among the intermediate vertices along with two pendant vertices forms an EQED-set. Since $|\text{deg}(u) - \text{deg}(v)| \leq 1$ where $uv \in E(P_n)$ for all $u \in D$ and $v \in V(P_n) - D$ and $\exists$ an eccentric vertex $u \in D$ for every $v \in V(P_n) - D$. For $P_n$ where $n = 3k$ and $k > 2$, number of vertices of $P_{3k-1}, P_{3k}, P_{3k+1}$ are same. Every minimum equitable eccentric domination set of $D$ contains $\left\lceil \frac{n+1}{3} \right\rceil + 1$ number of vertices.

**Theorem 3.3.** For star graph $S_n$,

$$\gamma_{eqed}(S_n) = \begin{cases} 2, & \text{if } n = 3 \\ 0, & \text{if } n \neq 3 \end{cases}$$

**Proof.** Case(i): If $n = 3$, then the star graph $S_3$ is isometric to $P_3$. From the theorem-3.2 $\gamma_{eqed}(P_3) = \gamma_{eqed}(S_3) = 2$.

Case(ii): If $n \neq 3$ then $S_n$ is of the form $S_4, S_5, S_6, \ldots$. For any graph $S_n$ where $n \neq 3$, there can be many dominating sets and eccentric dominating sets but we cannot find a EQED-set because of the central vertex $v_i$ of the star graph has degree $\geq 3$. The degree of every pendant vertex $u$ of a star graph is 1, $\text{deg}(u) = 1$, $u \in V(S_n) - \{v_i\}$. The degree of central vertex $v_i$ of a star graph is given by $\text{deg}(v_i) = n - 1$. Since, central vertex $v_i \in V(S_n)$ then either $v_i \in D$ or $v_i \in V(S_n) - D$. Therefore $|\text{deg}(v_i) - \text{deg}(u)| > 1$ always which doesnot satisfy the condition to be a EQED-set. Hence $\gamma_{eqed}(S_n) = 0$ where $n \neq 3$. 

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Theorem 3.4. For cycle graph $C_n$ where $n \geq 3$,

$$\gamma_{eqed}(C_n) = \begin{cases} 
\left\lfloor \frac{n}{2} \right\rfloor, & \text{if } n \text{ is even} \quad \forall \ n \geq 4 \\
\left\lceil \frac{n}{3} \right\rceil + 1, & \text{if } n \text{ is odd } \& n = 3k \forall \ k = 1, 3, 5, 7, \ldots \\
\left\lceil \frac{n}{3} \right\rceil, & \text{otherwise}
\end{cases}$$

Proof. Case(i): If $n'$ is even and $n \geq 4$. Let the cycles $C_n$ be of the form $C_4, C_6, C_8, C_{10}, \ldots C_{2n}$. In an even cycle if $u \in V(C_n)$ the eccentric vertex of $u$, $E(u) = \{v\}$ is always placed at a distance of $\frac{n}{2}$ edges from it and every vertex has a unique eccentric vertex to form the first eccentric dominating set. The set $D$ must contain $\frac{n}{2}$ vertices in such a way that for every $v \in D$ then $E(v) \not\ni D$ or for some $u \in V(C_n) - D$, $E(u) \not\ni V(C_n) - D$. Then if the vertex $u$ and $E(u) \in D$ then we cannot construct a eccentric dominating set. Further if we reduce the cardinality of $D$ to less than $\frac{n}{2}$ we will have $u$ and $E(u)$ in $V - D$. Therefore $D$ must contain $\frac{n}{2}$ vertices with all the unique eccentric vertices in $V - D$. Then for any $u \in V - D \exists$ a vertex $v \in D$ such that $|\deg(u) - \deg(v)| \leq 1$ where $uv \in E(C_n)$ for every vertex $v_i \in C_n$, $\deg(v_i) = 2$. Therefore $|\deg(u) - \deg(v)| = |2 - 2| = 0$. Therefore $\gamma_{eqed}(C_n) = \frac{n}{2}$.

Case(ii): Now we have the odd cycles of the form $C_3, C_6, C_9, C_{15}, C_{21}, \ldots C_{3k}$. Every vertex $u \in V(C_n)$ has two eccentric vertices $v_i, v_j$ such that $E(u) = \{v_i, v_j\}$. The eccentric vertices $v_i, v_j$ will always be adjacent i.e., $v_i, v_j \in E(C_n)$. $v_i, v_j$ are placed at a distance of $\frac{n-1}{2}$ edges from $u$. Since every vertex $u$ can dominate its adjacent vertices $v, w$. $\frac{n}{3}$ set of vertices form a dominating set of a cycle. The dominating set $D = \{v_i, v_j, v_n\}$ forms the EQED-set such that no eccentric vertices of $v_i \in D$ are in $D$. Then $\forall v \in V(C_n) - D \exists$ a vertex $u \in D \ni |\deg(u) - \deg(v)| = |2 - 2| = 0$. Therefore $\gamma_{eqed}(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

Case(iii): If $n = 3k + 1$ where $k$ is even. The cycles are of the form $C_7, C_{13}, C_{19}, \ldots, C_{3k+1}$ and if $n = 3k + 1$ where $k$ is odd, the cycles are of the form $C_5, C_{11}, C_{17}, \ldots, C_{3k+2}$. Totally we have $C_5, C_7, C_{11}, C_{13}, C_{17}, C_{19}, \ldots, C_{3k+1}, C_{3k+2}$. Similar to case(ii) every vertex $v_i \in V(C_n)$ has two eccentric vertices $v_l, v_m$. $E(v_l) = \{v_l, v_m\}$ such that $v_l$ and $v_m$ are adjacent i.e., $v_l, v_m \in E(C_n)$. Eccentric vertex $v_l$ and $v_m$ of $v_i$ are placed at a distance of $\frac{n-1}{2}$ from $v_i$. If $n = 3k$ we get $3, 9, 15, 21, \ldots$ which are the multiples of 3 we get a whole number which forms the cardinality of a EQED-set as proved in case(ii). But when $n = 3k + 1$ or $n = 3k + 2$ then $n = 5, 7, 11, 13, 17, 19, \ldots 3k + 1, 3k + 2$ which are not multiples of 3 we get a fraction value and also we are left out with a vertex which is to be dominated. Therefore the cardinality of the EQED-set of a cycles of the form $C_{3k+1}, C_{3k+2}$ increases by 1. Hence $\gamma_{eqed}(C_n) = \left\lceil \frac{n}{3} \right\rceil + 1$.

Theorem 3.5. Every EQED-set in a wheel graph $W_n, n \geq 6$ contains the central vertex.
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**Proof.** Let \( v_1 \) be the central vertex of the wheel graph \( W_n, n \geq 6 \) then \( \text{deg}(v_1) = n - 1 = \Delta(W_n) \). The degree of any non-central vertex \( u \in V(W_n) \) is \( \text{deg}(u) = 3 = \delta(W_n) \).

Suppose the central vertex \( v_1 \in V(W_n) - D, u \in D \) and \( D \) is an minimal eccentric dominating set we need to check for the condition of equitable domination then for \( v_1 \in V(W_n) - D \) and \( u \in D \), we have \( uv_1 \in E(W_n) \)

\[
\begin{align*}
|\text{deg}(v_1) - \text{deg}(u)| &= |\Delta(G) - \delta(G)| \\
|\text{deg}(v_1) - \text{deg}(u)| &= |(n - 1) - 3| \\
|\text{deg}(v_1) - \text{deg}(u)| &= |n - 4| \text{ where } n \geq 6 \\
|\text{deg}(v_1) - \text{deg}(u)| &> 1. \text{ which is a contradiction.}
\end{align*}
\]

Therefore the central vertex \( v_1 \) must belong to \( D \), if the set \( D \) is a equitable eccentric dominating set of \( W_n \).

**Theorem 3.6.** Let \( W_n \) be a wheel graph where \( n \geq 5 \) then EQED-set contains more than one vertex.

**Proof.** In any wheel graph \( W_n \) where \( n \geq 5 \). If the set \( D \subseteq V(W_n) \) contains the central vertex \( v_1 \) then \( D \) forms a dominating set as \( \text{deg}(v_1) = n - 1 = \Delta(G) \). But the eccentric vertices of a central vertex \( v_1 \) is given by \( E(v_1) = V - \{v_1\} \) and the eccentric vertex of any non-central vertex \( u \) is given by \( E(u) = V - N[u] \). Therefore there is no eccentric dominating or equitable eccentric dominating set of cardinality 1 for \( W_n \) where \( n \geq 5 \).

**Theorem 3.7.** For wheel graph \( W_n \), where \( n \geq 4 \) we have

\[
\gamma_{eqed}(W_n) = \begin{cases} 
1, & \text{if } n = 4 \\
4, & \text{if } n = 6 \\
\left\lceil \frac{n}{2} \right\rceil, & \text{if } n \text{ is odd and } n \geq 5 \\
\left\lceil \frac{n+1}{2} \right\rceil + 1, & \forall \ n \geq 8 \text{ and } n \text{ is even}
\end{cases}
\]

**Proof.** Case(i): If \( n = 4 \), \( W_4 \) is isometric to \( K_4 \), then by theorem-3.1 \( \gamma_{eqed}(W_4) = \gamma_{eqed}(K_4) = 1 \).

Case(ii): If \( n = 6 \), in a wheel graph \( W_6 \), there are no eccentric dominating sets of cardinality 1 or 2. Therefore we do not get an EQED-set of cardinality 1 or 2. There are sets of cardinality 3 which are eccentric dominating sets. But they do not form an EQED-set as the central vertex should not be present in \( V - D \). Since the degree of central vertex \( v_i \) is \( \text{deg}(v_i) = n - 1 = 5 \) and degree of any other non-central vertex is \( \text{deg}(v_j) = 3 \). Therefore \( |\text{deg}(v_i) - \text{deg}(v_j)| = 2 > 1 \) and in other cases if \( v_i \notin V - D \) then we find a combination of vertices of 3 cardinality which are eccentric dominating set but they don’t form an INED-set since for some vertex \( v \in V - D \) there is no vertex \( u \in D \) such that \( u, v \notin E(W_6) \). But we find a EQED-set with cardinality 4 as we have the central vertex in \( D \). Then \( |\text{deg}(v_i) - \text{deg}(v_j)| \leq 1, (v_i, v_j) \in E(W_6) \) where \( v_i \in D \) and \( v_j \in V(W_6) - D \).
Therefore $\gamma_{eqed}(W_6) = 4$.

Case(iii): If $n$ is odd and $n \geq 5$ we have the wheel graph of order $W_5, W_7, W_9, W_{11}, \ldots$
If $v \in V(W_n)$ then $|E(v)| = n - 4$. There will always be $n - 4$ vertices which
form the eccentric vertex $E(v)$ for every vertex $v$. And for any wheel graph where
'nn' is odd. The set $D \subseteq V(W_n)$ forms an eccentric dominating sets only when
$|D| = \left\lceil \frac{n}{2} \right\rceil$. Then for every $v \in V(W_n) - D$ there exists a vertex $u \in D$
such that $|\text{deg}(u) - \text{deg}(v)| \leq 1$ and $(u, v) \in E(W_n)$. Therefore $\gamma_{eqed}(W_n) = \left\lceil \frac{n}{2} \right\rceil$.

Case(iv): The wheel graph $W_n$ where $n$ is even and $n \geq 8$ has $n - 4$ eccentric vertices. We have wheel graphs $W_8, W_{10}, W_{12}, \ldots$ For every vertex $v \in V(W_n)$,
$|E(v)| = n - 4$. From theorem-3.6.3.5, $\gamma_{eqed}(W_n) \neq 1$ and the central vertex
$v_1 \in D$ then $D$ contains other vertices of $W_n$ where cardinality of $D$ is of the form
$\left\lceil \frac{n+1}{2} \right\rceil + 1$. For every $v \in V - D$ there exists a vertex $u \in D$
such that $E(v)$ lies in $D$ and $|\text{deg}(u) - \text{deg}(v)| \leq 1$ such that there exists an edge between $u$ and $v$.
Therefore $\gamma_{eqed}(W_n) = \left\lceil \frac{n+1}{2} \right\rceil + 1$.

**Theorem 3.8.** An EQED-set $D$ is a minimal EQED-set if one of the following
conditions holds,
1. For every vertex $u$ in $V - D$ there does not exists $v$ in $D$ such that $E(u) = \{v\}$
ie, $u$ has no eccentric vertex in $D$.
2. There exists some $u \in V - D$ such that $N(u) \cap D = \{v\}$, $E(u) \cap D = \{v\}$
and $|d(u) - d(v)| \leq 1$ where $uv \in E(G)$.

**Proof.** Suppose $D$ is a minimal EQED-set of $G$. Then for every vertex $v$ in
$D, D - \{v\}$ is not an EQED-set. Thus there exists some vertex $u$ in $V - D \cup \{v\}$
which is not dominated by any vertex in $D - \{v\}$ or there exists $u \in V - D \cup \{v\}$
such that $u$ does not have an eccentric vertex in $D - \{v\}$ ie, $E(u) \neq D - \{v\}$
or $|d(u) - d(v)| \neq 1$ or $uv \notin E(G)$. : : The concept of equitable condition
does not hold. Case(i): If $v = u$ then $u$ does not have an eccentric vertex in $D$ ie,
$E(u) \neq D$. Case(ii): If $v \neq u$, (a) If $u \in V - D$ and $u$ is not dominated by $D - \{v\}$,
but dominated by $D$ then $u$ is adjacent to only $v$ in $D$ ie, $N(u) \cap D = \{v\}$. (b) If
$u \in V - D$ and $u$ does not have an eccentric vertex in $D - \{v\}$ but $u$ has an eccentric
vertex in $D$. Thus $v$ is the only eccentric vertex of $u$ in $D$ ie, $E(u) \cap D = \{v\}$.
(c) If $u \in V - D$ and $|d(u) - d(x)| \neq 1$ or $ux \notin E(G)$ where $x \in D - \{v\}$ but
$|d(u) - d(x)| \leq 1$ and $uv \in E(G)$. Conversely, Suppose $D$ is an EQED-set and
for each $v \in D$, one of the two conditions holds. Now we show that $D$ is a min-
imal EQED-set. Suppose $D$ is not an minimal EQED-set ie, there exists a vertex
$v \in D$ such that $D - \{v\}$ is an EQED-set. Hence $v$ is adjacent to at least one
vertex $x$ in $D - \{v\}$, $v$ has an eccentric vertex in $D - \{v\}$ ie, $E(v) \subset D - \{v\}$
and $|d(u) - d(x)| \leq 1$ where $ux \in E(G)$. : : Equitable condition holds and EQED-set
exists. Also if $D - \{v\}$ is an EQED-set, then every vertex $u$ in $V - D$ is ad-
joint to at least one vertex $x$ in $D - \{v\}$, $u$ has an eccentric vertex in $D - \{v\}$
ie, $E(u) \subset D - \{v\}$ and $|d(u) - d(x)| \leq 1$ and $ux \in E(G)$. Therefore condition-(2)
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does not hold. Hence neither condition-(1) nor (2) holds, which is a contradiction to our assumption. Hence for each \( v \in D \) one of the 2 conditions holds.

The equitable eccentric dominating set, \( \gamma_{eqed}(G) \), upper equitable eccentric dominating set and \( \Gamma_{eqed}(G) \) of standard graphs are tabulated.

| Graph       | Figure   | D - Minimum EQED set. \( |D| = \gamma_{eqed}(G) \) | \( \gamma_{eqed}(G) \) | S - Upper EQED set. \( |S| = \Gamma_{eqed}(G) \) | \( \Gamma_{eqed}(G) \) |
|-------------|----------|-----------------------------|----------------|--------------------------------|----------------|
| Diamond graph | ![Diamond Graph](image) | \( \{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_3, v_4\} \) | 2 | \( \{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_3, v_4\} \) | 2 |
| Tetrahedral graph | ![Tetrahedral Graph](image) | \( \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\} \) | 1 | \( \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\} \) | 1 |
| Claw graph | ![Claw Graph](image) | Does not exist | 0 | Does not exist | 0 |
| Paw graph | ![Paw Graph](image) | \( \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\} \) | 2 | \( \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\} \) | 2 |
| Bull graph | ![Bull Graph](image) | \( \{v_1, v_2, v_3\}, \{v_1, v_3, v_4\}, \{v_1, v_2, v_4\} \) | 3 | \( \{v_1, v_2, v_3\}, \{v_1, v_3, v_4\}, \{v_1, v_2, v_4\} \) | 3 |
| Butterfly graph | ![Butterfly Graph](image) | \( \{v_1, v_2, v_3\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4\}, \{v_3, v_4, v_5\} \) | 3 | \( \{v_1, v_2, v_3\}, \{v_1, v_3, v_5\}, \{v_2, v_3, v_4\}, \{v_3, v_4, v_5\} \) | 3 |
| Banner graph | ![Banner Graph](image) | \( \{v_1, v_2, v_5\}, \{v_1, v_3, v_5\}, \{v_2, v_3, v_5\}, \{v_2, v_4, v_5\}, \{v_3, v_4, v_5\} \) | 3 | \( \{v_1, v_2, v_5\}, \{v_1, v_3, v_5\}, \{v_2, v_3, v_5\}, \{v_2, v_4, v_5\}, \{v_3, v_4, v_5\} \) | 3 |
| Graph         | Figure | D - Minimum EQED set. $|D| = \gamma_{eqed}(G)$ | $\gamma_{eqed}(G)$ | S - Upper EQED set. $|S| = \Gamma_{eqed}(G)$ | $\Gamma_{eqed}(G)$ |
|--------------|--------|------------------------|---------------------|------------------------|---------------------|
| Fork graph   | ![Fork Graph](image) | $\{v_1, v_2, v_3, v_4\}$, $\{v_1, v_2, v_4, v_5\}$, $\{v_1, v_3, v_4, v_5\}$. | 4                   | $\{v_1, v_2, v_3, v_4\}$, $\{v_1, v_2, v_4, v_5\}$, $\{v_1, v_3, v_4, v_5\}$. | 4                   |
| (3,2)-Tadpole graph | ![Tadpole Graph](image) | $\{v_1, v_4\}$, $\{v_4, v_5\}$. | 2                   | $\{v_1, v_2, v_3, v_5\}$. | 4                   |
| Kite graph   | ![Kite Graph](image) | $\{v_1, v_2, v_4\}$, $\{v_1, v_3, v_4\}$, $\{v_2, v_3, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_3, v_4, v_5\}$. | 3                   | $\{v_1, v_2, v_4\}$, $\{v_1, v_3, v_4\}$, $\{v_2, v_3, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_3, v_4, v_5\}$. | 3                   |
| (4,1)-Lollipop graph | ![Lollipop Graph](image) | $\{v_1, v_4\}$, $\{v_2, v_4\}$, $\{v_3, v_4\}$, $\{v_3, v_5\}$, $\{v_4, v_5\}$. | 2                   | $\{v_1, v_2, v_4\}$, $\{v_1, v_3, v_4\}$, $\{v_3, v_4\}$. | 2                   |
| House graph  | ![House Graph](image) | $\{v_2, v_4\}$, $\{v_3, v_5\}$. | 2                   | $\{v_1, v_2, v_3\}$, $\{v_1, v_4, v_5\}$. | 3                   |
| House X graph | ![House X Graph](image) | $\{v_1, v_2\}$, $\{v_1, v_3\}$, $\{v_2, v_3\}$, $\{v_1, v_4\}$, $\{v_2, v_5\}$. | 2                   | $\{v_1, v_2\}$, $\{v_1, v_3\}$, $\{v_2, v_5\}$. | 2                   |
| Gem graph    | ![Gem Graph](image) | $\{v_3, v_2\}$. | 2                   | $\{v_1, v_2\}$, $\{v_2, v_3, v_4\}$, $\{v_3, v_4, v_5\}$. | 3                   |
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<p>| Graph                     | Figure | D - Minimum EQED set. $|D| = \gamma_{eqed}(G)$ | $\gamma_{eqed}(G)$ | S - Upper EQED set. $|S| = \Gamma_{eqed}(G)$ | $\Gamma_{eqed}(G)$ |
|---------------------------|--------|----------------------|-------------------|----------------------|-------------------|
| Dart graph                | <img src="#" alt="Figure D" /> | ${v_2, v_4}$ | 2 | ${v_1, v_2, v_3, v_5}$ | 4 |
| Cricket graph             | <img src="#" alt="Figure S" /> | ${v_1, v_3, v_4, v_5}, {v_2, v_3, v_4, v_5}$ | 4 | ${v_1, v_3, v_4, v_5}, {v_2, v_3, v_4, v_5}$ | 4 |
| Pentatope graph           | <img src="#" alt="Figure P" /> | ${v_1}, {v_2}, {v_3}, {v_4}, {v_5}$ | 1 | ${v_1}, {v_2}, {v_3}, {v_4}, {v_5}$ | 1 |
| Johnson solid skeleton-12 | <img src="#" alt="Figure J" /> | ${v_1, v_2}, {v_1, v_3}, {v_1, v_4}, {v_1, v_5}, {v_2, v_3}, {v_2, v_4}, {v_2, v_5}, {v_3, v_4}, {v_3, v_5}$ | 2 | ${v_1, v_2}, {v_1, v_3}, {v_1, v_4}, {v_1, v_5}, {v_2, v_3}, {v_2, v_4}, {v_2, v_5}, {v_3, v_4}, {v_3, v_5}$ | 2 |
| Cross graph               | <img src="#" alt="Figure C" /> | ${v_1, v_2, v_3, v_4, v_6}, {v_1, v_2, v_3, v_4, v_6}, {v_1, v_2, v_3, v_4, v_6}$ | 5 | ${v_1, v_2, v_3, v_4, v_6}, {v_1, v_2, v_3, v_4, v_6}, {v_1, v_2, v_3, v_4, v_6}$ | 5 |
| Net graph                 | <img src="#" alt="Figure N" /> | ${v_1, v_2, v_3, v_6}, {v_1, v_2, v_4, v_6}, {v_1, v_2, v_5, v_6}$ | 4 | ${v_1, v_2, v_3, v_6}, {v_1, v_2, v_4, v_6}, {v_1, v_2, v_5, v_6}$ | 4 |
| Fish graph                | <img src="#" alt="Figure F" /> | ${v_2, v_3, v_4}, {v_3, v_4, v_5}$ | 3 | ${v_1, v_2, v_4, v_5}$ | 5 |</p>
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<th>Figure</th>
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<th>$\gamma_{\text{eqed}}(G)$</th>
<th>$\text{S - Upper EQED set.} [S] = \Gamma_{\text{eqed}}(G)$</th>
<th>$\Gamma_{\text{eqed}}(G)$</th>
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4 Conclusions

Inspired by eccentric dominating set and equitable dominating set we introduce the equitable eccentric dominating set. We find minimum equitable eccentric dominating set, minimum equitable eccentric domination number $\gamma_{\text{eqed}}(G)$, upper equitable eccentric dominating set and upper equitable eccentric domination number $\Gamma_{\text{eqed}}(G)$ of different standard graphs. We have discussed the properties and proved theorems related to equitable eccentric dominating set of family of graphs.

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References


